

# RATE PROBLEM SOLVING

## LESSON 2.4



Solve problems using equivalent rates and unit rates.

The fractions  $\frac{2}{4}$ ,  $\frac{3}{6}$ ,  $\frac{4}{8}$ , and  $\frac{5}{10}$  can be written in simplest form as  $\frac{1}{2}$ . These are examples of **equivalent fractions**. Equivalent fractions are fractions with the same value. Since ratios and rates can be written as fractions, they also can be written in many equivalent forms.

### EXPLORE!

Below are 10 rates.

$$\frac{\$8.00}{4 \text{ tickets}}$$

$$\frac{\$10.00}{2 \text{ tickets}}$$

$$\frac{\$16.00}{2 \text{ tickets}}$$

$$\frac{\$12.00}{4 \text{ tickets}}$$



$$\frac{\$5.00}{1 \text{ ticket}}$$

$$\frac{\$12.00}{6 \text{ tickets}}$$

$$\frac{\$24.00}{4 \text{ tickets}}$$

$$\frac{\$6.00}{1 \text{ ticket}}$$

$$\frac{\$9.00}{3 \text{ tickets}}$$

$$\frac{\$24.00}{3 \text{ tickets}}$$

**Step 1:** Each rate has the same units. Write the units for the rates. (\_\_\_\_\_ per \_\_\_\_\_)

**Step 2:** Which of the above rates are already written as unit rates?

**Step 3:** There are five pairs of equivalent rates. One is given below. Find the four other pairs. Write the pairs next to one another with an equals sign between the two rates.

1.  $\frac{\$12.00}{4 \text{ tickets}} = \frac{\$9.00}{3 \text{ tickets}}$       2.      3.      4.      5.

**Step 4:** Explain how you figured out which rates were equivalent.

**Step 5:** The price for a ticket to a jazz concert was \$14. Write 5 equivalent rates using the unit rate of \$14 per ticket.

Problems involving rates can be solved using two different methods. You can use equivalent fractions or unit rates.

### SOLVING PROBLEMS INVOLVING RATES

#### Using Equivalent Fractions

1. Write the two rates with an equals sign (=) between them.
2. Identify what you need to multiply the numerator or denominator of the complete rate by to write an equivalent rate on the other side of the equals sign.

#### Using Unit Rates

1. Find the unit rate for the known rate.
2. Multiply the unit rate by the known quantity.

### EXAMPLE 1

Complete each equivalent rate.

a.  $\frac{24 \text{ miles}}{1 \text{ gallon}} = \frac{\text{miles}}{6 \text{ gallons}}$

b.  $\frac{\$ 6.00}{4 \text{ liters}} = \frac{\$}{32 \text{ liters}}$

#### SOLUTIONS

- a. Find the factor from one denominator to the other.

$$\frac{24 \text{ miles}}{1 \text{ gallon}} = \frac{\text{miles}}{6 \text{ gallons}}$$

$\swarrow \times 6 \searrow$

Multiply the numerator by the same factor to complete the equivalent rate.

$$\frac{24 \text{ miles}}{1 \text{ gallon}} = \frac{144 \text{ miles}}{6 \text{ gallons}}$$

$\swarrow \times 6 \searrow$

- b. Find the factor from one denominator to the other.

$$\frac{\$ 6.00}{4 \text{ liters}} = \frac{\$}{32 \text{ liters}}$$

$\swarrow \times 8 \searrow$

Multiply the numerator by the same factor to complete the equivalent rate.

$$\frac{\$ 6.00}{4 \text{ liters}} = \frac{\$ 48.00}{32 \text{ liters}}$$

$\swarrow \times 8 \searrow$

### EXAMPLE 2

Nigel paid \$3.60 for 30 copies of his flyer. Use a unit rate to determine the cost to make 80 copies of his flyer.

#### SOLUTION

Write the rate as a fraction.

$$\frac{\$3.60}{30 \text{ copies}}$$

Find the unit rate.

$$\frac{\$3.60}{30 \text{ copies}} = \frac{\$0.12}{1 \text{ copy}}$$

$\swarrow \div 30 \searrow$   
 $\nwarrow \div 30 \nearrow$

Multiply the cost per copy by the number of copies.

$$\$0.12 \times 80 = \$9.60$$

Nigel will pay \$9.60 to make 80 copies of his flyer.

### EXAMPLE 3

Tom buys some apples at a local fruit stand. The fruit stand charges \$3.00 for every 2 pounds. Find the price Tom pays for 12 pounds of apples.



### SOLUTION

#### METHOD 1 ~ Equivalent Fractions

Write the rate as a fraction.

$$\frac{\$3.00}{2 \text{ lbs}}$$

Write a second fraction with a denominator of 12 pounds.

$$\frac{\$3.00}{2 \text{ lbs}} = \frac{\$}{12 \text{ lbs}}$$

The new denominator is 6 times the original denominator. Multiply the numerator by 6.

$$\begin{array}{ccc} & \nearrow \times 6 & \searrow \\ \frac{\$3.00}{2 \text{ lbs}} & = & \frac{\$18.00}{12 \text{ lbs}} \\ & \searrow \times 6 & \nearrow \end{array}$$

Tom pays \$18 for 12 pounds of apples.

#### METHOD 2 ~ Unit Rates

Write the rate as a fraction.

$$\frac{\$3.00}{2 \text{ lbs}}$$

Find the unit rate.

$$\begin{array}{ccc} & \nearrow \div 2 & \searrow \\ \frac{\$3.00}{2 \text{ lbs}} & = & \frac{\$1.50}{1 \text{ lb}} \\ & \searrow \div 2 & \nearrow \end{array}$$

Multiply the cost per pound (\$1.50) by the number of pounds.

$$\$1.50 \times 12 = \$18.00$$

Tom pays \$18 for 12 pounds of apples.

## EXERCISES

Complete each equivalent rate.

1.  $\frac{\$3.00}{1 \text{ gallon}} = \frac{\$}{10 \text{ gallons}}$

2.  $\frac{3 \text{ miles}}{1 \text{ hour}} = \frac{\text{miles}}{8 \text{ hours}}$

3.  $\frac{60 \text{ words}}{2 \text{ minutes}} = \frac{\text{words}}{14 \text{ minutes}}$

4.  $\frac{3 \text{ kilometers}}{1 \text{ hour}} = \frac{\text{kilometers}}{3 \text{ hours}}$

5.  $\frac{25 \text{ miles}}{1 \text{ gallon}} = \frac{200 \text{ miles}}{\text{gallons}}$

6.  $\frac{12 \text{ jobs}}{5 \text{ days}} = \frac{48 \text{ jobs}}{\text{days}}$

Use equivalent rates to complete each problem.

7. Felicia drove 120 miles in 3 hours. At this rate, how far will she drive in 6 hours?

8. Marcus burns 9 calories per minute when running. How long will he need to run to burn 270 calories? Show all work necessary to justify your answer.

9. Henry paid \$60 for 5 people to attend a play on Broadway. Next month, 15 people in his class would like to go. If the cost is the same per ticket, how much will Henry pay for 15 people to attend next month?



Find each unit rate. Round to the nearest hundredth, if necessary.

10.  $\frac{8 \text{ feet}}{2 \text{ minutes}}$

11.  $\frac{\$4.00}{10 \text{ pencils}}$

12.  $\frac{70 \text{ miles}}{3 \text{ gallons}}$

13.  $\frac{12 \text{ meters}}{48 \text{ seconds}}$

14.  $\frac{105 \text{ words}}{2 \text{ minutes}}$

15.  $\frac{\$8.00}{12 \text{ books}}$

Use a unit rate to complete each problem.

16. Jimmy's new car went 204 miles using 12 gallons of gas. At this rate, how many miles can he travel using 5 gallons of gas? Show all work necessary to justify your answer.
17. Patrick went to the store to buy a seedless watermelon. It was on sale for \$0.88 for every 2 pounds. He bought an 11 pound watermelon. How much did Patrick pay for the watermelon?
18. Denise filled her wading pool using her garden hose. The pool filled at a rate of 7 gallons every 2 minutes. She left the water on for 9 minutes. How many gallons of water were in the wading pool?

Use equivalent fractions or unit rates to solve each problem.

19. Aaron walked 8 miles in 2 hours. To determine how far he could walk at this rate in 6 hours, Aaron used a unit rate and showed the following work. Show another way to find how far Aaron can walk at that rate in 6 hours.

$$\frac{8 \text{ miles}}{2 \text{ hours}} = \frac{4 \text{ miles}}{1 \text{ hour}}$$

$$4 \text{ miles per hour} \times 6 \text{ hours} = 24 \text{ miles}$$

20. Josh spent \$4.40 for 4 candy bars at the student store. How much would he pay for 7 candy bars at the student store?



21. Miranda's mom sent her to the grocery store with \$20.00. She bought 2 pounds of roast beef, 3 pounds of apples, 1 loaf of bread and 1 gallon of milk. She could buy anything else at the store she wanted with the remaining money. Use the prices below to determine if she had enough money to purchase one cookie and one bag of popcorn. Show all work necessary to justify your answer.

Roast beef: \$5.00 per pound

Apples: \$2.50 for 2 pounds

Bread: \$2.00 per loaf

Milk: \$2.50 per gallon

Juice Box: \$0.50 per box

Cookie: \$1.50 for 2 cookies

Candy bar: \$1.00 per candy bar

Popcorn: \$1.25 per bag

## REVIEW

Complete each conversion. Show all work necessary to justify your answer.

22. 3 kilometers = \_\_\_\_\_ meters

23. 35 millimeters = \_\_\_\_\_ centimeters

24. 42 inches = \_\_\_\_\_ feet

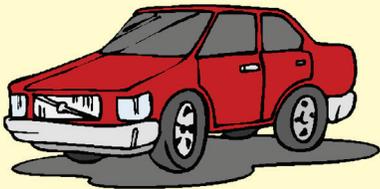
25. 5 yards = \_\_\_\_\_ feet

26. Lonnie spent \$9.30 on 3 small cakes. Find the price per cake.

27. Jeff walked 27 miles in 6 hours. Find his speed in miles per hour.

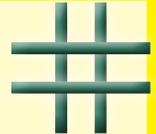
28. There were eight boys and 20 girls at a party when it started. When it ended, there were 12 boys at the party but the same ratio of boys to girls as when the party began. How many girls were at the party when it ended? Use mathematics to justify your answer.

### TIC-TAC-TOE ~ GAS MILEAGE



The gas mileage a car gets is the ratio of the miles a car has driven to the number of gallons of gas used.

$$\text{Gas Mileage} = \frac{\text{miles driven}}{\text{gallons of gas used}}$$



*Example:* A car traveled 235 miles using 12 gallons of gas. Its gas mileage is  $\frac{235 \text{ miles}}{12 \text{ gallons}}$ . The rate for gas mileage is usually written as a decimal rounded to the nearest tenth.

In this case,  $\frac{235 \text{ miles}}{12 \text{ gallons}} = 19.58\bar{3} \approx 19.6$  miles per gallon.

**Step 1:** Record the gas mileage of your family car. Do this by writing down the number of miles driven since the last fill-up and the amount of gas needed to fill up the tank at the gas station.

**Step 2:** Record the gas mileage of your family car one more time to compare the two rates.

**Step 3:** What is the estimated gas mileage for your family car based on your data?

**Step 4:** Research your car to find out what the manufacturer says the gas mileage should be.

**Step 5:** Research to find which cars have the best gas mileage (most miles per gallon). Create a list of the top five cars.

# TIC-TAC-TOE ~ TYPING



How many words per minute can you type? Use a timer or ask a friend to time you as you type the following story.

Sally went to the store with her mother and brother and bought some milk, carrots, onions, salad dressing and tomatoes. Next, Sally's mom took her to the dentist and the dry cleaners. Sally wanted to go home and play with her friends. Finally, Sally's mom was done with errands for the day. She took Sally to the park to play with her friends. Sally's friend, Tom, asked her what she had done that day. She told Tom she went to the store, the dentist and the dry cleaners. Tom reminded her that they had soccer practice in the evening. Sally told him she would see him at practice. She left for home to get ready.

**Step 1:** Type the entire 115 word paragraph and time yourself. Record the number of seconds it took you to type the passage. Also record the number of errors you made. Keep typing the passage until you make fewer than 5 errors. If this happens on your first try, type faster and see how many errors you make. Type the passage and record the information at least three times.

Attempts	Time (sec)	Number of Errors
1		
2		
3		

**Step 2:** Convert the time it took you to type the passage from seconds to minutes. Round to the nearest hundredth.

**Step 3:** What was your fastest typing rate as a unit rate of words per minute?

**Step 4:** What was your fastest typing rate with fewer than 5 errors?

**Step 5:** How long would it take you to type a 1-page paper with 460 words at your fastest rate? Show all work necessary to justify your answer.