DRAWING GEOMETRIC SHAPES

LESSON 1.5

Draw geometric shapes with given conditions.

What makes a triangle? Three angles and three sides form a triangle. Are there any limitations when trying to draw a triangle? Will any three lengths work? Will any three angles form a triangle? If two triangles have the same angle measures, are they congruent? If the two triangles have the same side lengths, are they congruent? You will discover the answers to these questions in the following two-part **Explore!**.

EXPLORE! **KNOWING THREE MEASURES**

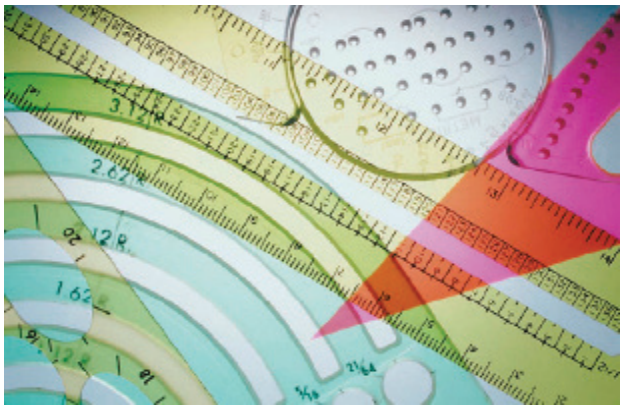
Part 1: Three Angles

Step 1: Use a protractor and straightedge to draw a triangle with angles that are 90°, 20° and 70°.

Step 2: Write the angle measures inside each corresponding vertex. Cut out the triangle.

Step 3: Tape your triangle in the place your teacher has designated for this group of triangles.

Step 4: Next, use a protractor and straightedge to construct a triangle with two angles that are 125° and 20°. Determine the measure of the third angle. Write the angle measure inside each vertex of the triangle.



Step 5: Cut the triangle out and tape it in the place your teacher has designated for this set of triangles.

Step 6: Look at the first group of triangles. What do you observe about the size and shape of these triangles?

Step 7: Look at the second group of triangles. What do you notice about the angle measures, shape and size of each triangle?

Step 8: When you compare two triangles with three congruent angles, will the triangles always be unique? In other words, is there only one possible triangle that can be formed by given congruent angles? Be specific in terms of size and shape.

In ***Part 1*** of the **Explore!** you found that when three angles in one triangle are congruent to three angles in another triangle the two triangles may not be congruent. The two triangles have the same shape but may not be the same size. When two triangles have congruent angles and proportional sides they are called **similar**. It is also true that when two angles are congruent to two angles in another triangle the triangles are similar. When this occurs, the third angles in each triangle will also be congruent to each other.

Angle - Angle Similarity Rule

When two angles in one triangle are congruent to two angles in another triangle, the triangles will be similar.

EXPLORE! KNOWING THREE MEASURES

Part 2: Three Sides

Step 1: Cut out thin strips of paper in the following lengths: 4 *cm*, 6 *cm*, 7 *cm*, 10 *cm*, 12 *cm*, 13.5 *cm*, 15 *cm*, 24 *cm*.

Step 2: Copy tables like the ones below on a piece of paper.

Lengths that Form a Triangle

Short Side	Medium Side	Long Side

Lengths that DO NOT Form a Triangle

Short Side	Medium Side	Long Side

Step 3: Use the following sets of three lengths of strips of paper to try to form a triangle. Record the lengths in the appropriate table.

4 *cm*, 6 *cm*, 12 *cm* 7 *cm*, 10 *cm*, 15 *cm* 13.5 *cm*, 15 *cm*, 24 *cm* 7 *cm*, 10 *cm*, 24 *cm*

Step 4: Use the strips of paper to find two additional sets of strips of paper that form a triangle. Find two additional sets that do not form a triangle. Record each set of lengths in the appropriate table.

Step 5: In the table labeled "Lengths that Form a Triangle", calculate the sum of the short and medium sides of each triangle. What do you notice about the sum of the two shortest sides compared to the length of the longest side? Explain.

Step 6: Repeat **Step 5** for the table labeled "Lengths that DO NOT Form a Triangle". Explain what you notice about the sum of the lengths of the two shortest sides compared to the length of the longest side in this table.

Step 7: Using your conclusions in **Step 5** and **Step 6**, determine if the following lengths will form a triangle.

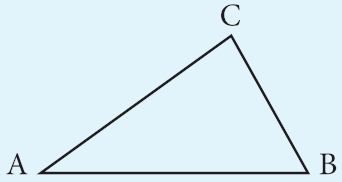
a. 5 *in*, 7 *in*, 14 *in* **b.** 12 *in*, 15 *in*, 23 *in* **c.** 8 *ft*, 8 *ft*, 8 *ft* **d.** 12.4 *m*, 15 *m*, 27.2 *m*

Step 8: Determine if lengths of 8 *cm*, 10 *cm* and 18 *cm* can form a triangle. Explain what you did to reach your conclusion.

Step 9: Write a rule about the lengths of the three sides of a triangle.

Side Length Inequality Rule

The sum of the lengths of any two sides of a triangle must be greater than the third side.



$AB + AC > BC$
$AC + CB > AB$
$AB + BC > AC$

When determining if three lengths will form a triangle it is only necessary to verify that the sum of the lengths of the two shortest sides is greater than the length of the longest side.

EXAMPLE 1

Determine if each set of side lengths can form a triangle.
a. 4 *cm*, 9 *cm*, 15 *cm*
b. 2 *in*, 2 *in*, 4 *in*
c. 14 *m*, 9.4 *m*, 8.3 *m*

Solutions

	a.	**b.**	**c.**
Write each inequality comparing the sum of the two shortest sides to the longest.	Is $4 + 9 > 15$?	Is $2 + 2 > 4$?	Is $8.3 + 9.4 > 14$?
Simplify.	$13 > 15$	$4 > 4$	$17.7 > 14$
Is the sum more than the longest side?	No	No	Yes

a. No, the lengths 4 *cm*, 9 *cm* and 15 *cm* cannot form a triangle.
b. No, the lengths 2 *in*, 2 *in* and 4 *in* cannot form a triangle.
c. Yes, the lengths 14 *m*, 9.4 *m* and 8.3 *m* can form a triangle.

EXAMPLE 2

A triangle has two sides that are 7 feet and 12 feet. Find the values the third side must be between.

Solution

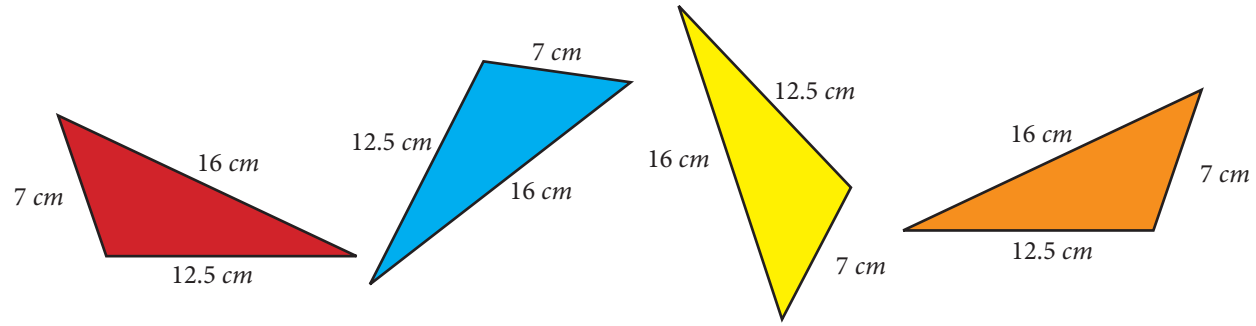
Write each inequality using the known side lengths.
Let x be the unknown side.

If x is the short or medium side: $7 + x > 12$
$x > 5$

If x is the longest side: $12 + 7 > x$
$19 > x$

The third side must be greater than 5, but less than 19.

Rachel and Jackson each drew a triangle with matching side lengths. When they finished they noticed that their triangles looked exactly the same. Both of them made another triangle with the same side lengths and all the triangles were congruent to one another. Their triangles are represented below.



Triangles are congruent when three sides in one triangle are congruent to the corresponding sides in another triangle.

Side-Side-Side Triangle Congruence

If all three sides in one triangle are the same length as the corresponding sides in another triangle, then the triangles are congruent.

EXERCISES

Determine if each statement is *ALWAYS*, *SOMETIMES* or *NEVER* true. Explain your reasoning.

1. The sum of the two shortest sides of a triangle must be less than the longest side.

2. If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

3. If three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent.

4. In every pair of congruent triangles, there are 3 sets of corresponding congruent sides and 3 sets of corresponding congruent angles.

5. If two figures are similar, then they are congruent.

6. If two angles in one triangle are congruent to two angles in another triangle, the two triangles are congruent.

Determine if each set of lengths will form a triangle.

7. 5 *cm*, 9 *cm*, 12 *cm*

8. 3.6 *m*, 7.2 *m*, 10.9 *m*

9. 14 *ft*, 23 *ft*, 19 *ft*

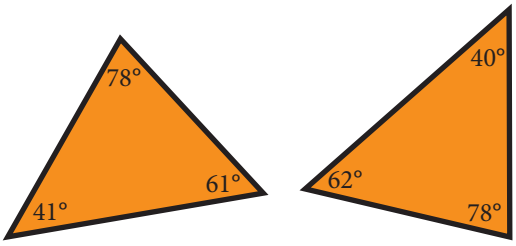
10. 8.5 *cm*, 8.5 *cm*, 8.5 *cm*

11. 38 *in*, 21 *in*, 13 *in*

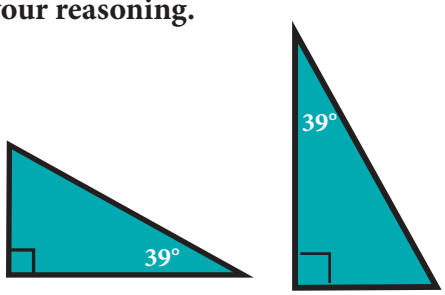
12. 1.5 *km*, 1.1 *km*, 0.4 *km*

Determine if each pair of triangles is similar. Explain your reasoning.

13.



14.



15. Construct two triangles with sides that are 5 *cm*, 3 *cm* and 7 *cm*. Label the first one ΔCAT and the second one ΔDOG. Are they congruent? Explain your reasoning.

16. A triangle has two sides that are 3 feet and 11 feet long. Which values below could be the length of the third side? Write all that apply.

5 feet 8 feet 9 feet 10.7 feet 14 feet 16.5 feet

Two sides of a triangle are given. Determine the lengths the third side must be between.

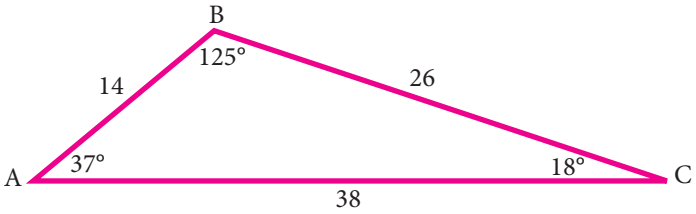
17. 12 inches, 17 inches

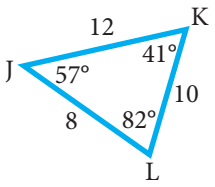
18. 2.5 *cm*, 4.7 *cm*

19. 42 *ft*, 32.5 *ft*

20. Manuel knows the longest side length of a triangle is 8 inches. The other side lengths are integer measures. List all possible combinations of the other two side lengths.

For Exercises 21–26, use the triangles below.





21. Is ΔABC congruent to ΔJKL? Explain your reasoning.

22. Is ΔABC similar to ΔJKL? Explain your reasoning.

23. Sketch a triangle that is similar to ΔABC but not congruent. Label each side and each angle with appropriate measures.

24. Sketch a triangle that is similar to ΔJKL but is not congruent. Label each side and each angle with appropriate measures.

25. Sketch ΔVWT making it congruent to ΔABC. Label each side and each angle with appropriate measures.

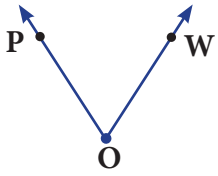
26. Sketch ΔMOP making it congruent to ΔJKL. Label each side and each angle with appropriate measures.

27. Prior to the start of a sailboat race, the judges must make sure all of the sails are the same shape. Explain how the judges can verify that each sail is the same shape without taking the sails down.

REVIEW

Write two possible names for each angle.

28.



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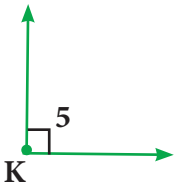

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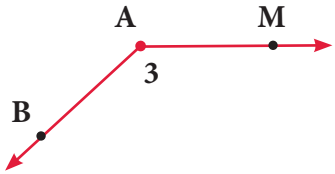


Sketch and label a diagram for each description.

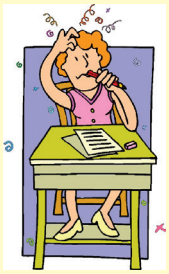
31. an acute angle

32. vertical angles with each angle measuring 40°

33. adjacent supplementary angles

34. complementary angles that are not adjacent

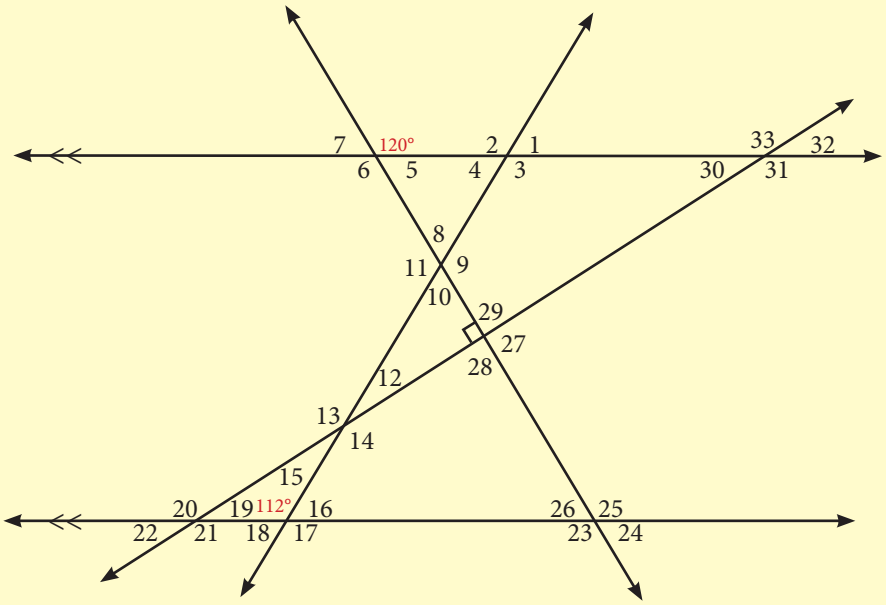
Tic-Tac-Toe ~ Puzzling Angles



Find the measure of the numbered angles in the diagram. You may need to gather additional information about angles before completing the puzzle.

a. Find what the sum of the angles in a triangle is.

b. Find what the angles of a quadrilateral add up to.



Tic-Tac-Toe ~ Crossword

Create a crossword puzzle using all of the vocabulary from **Block 1**. Make a blank master copy and an answer key.