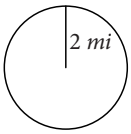
Materials Needed

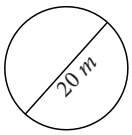
Compasses or circular objects, rulers, scissors

Warm-Up Problems

1) Find the circumference of each circle. Use 3.14 for π.

a) 

≈ 12.56 miles

b) 

≈ 62.8 meters

Simplify each expression.

2) $2^2 \cdot 5$ 20

3) $3^2 - 1^2$ 8

4) $4(10)^2$ 400

Explore!

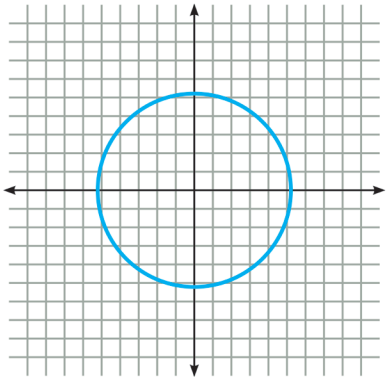
Step 1: Have compasses or circular objects for students to trace around available prior to class. Remind students that circles should have a minimum radius of 2 inches. To save time, provide pre-cut circles. Larger circles are preferable.

CCSS 7.G.4

AREA OF A CIRCLE

LESSON 2.5

Understand and use the circle area formula.



Maria and Greg are making circular coasters for a family dinner. They want the radius of each coaster to be 5 centimeters long. How many square centimeters will each coaster cover on a table?

Maria drew a coordinate plane. She sketched the circular coaster. She estimated the area by counting the number of squares inside the circle. She counted approximately 76 squares or parts of squares inside the drawing. Greg thought there might be a more accurate method to find the area of each coaster.

You will learn how to find the area of a circle using a formula in this lesson.

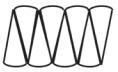
EXPLORE! **CIRCLE AREAS**

Step 1: Trace a circle onto a piece of paper or use a compass to draw a circle. The radius of the circle should be at least 2 inches.

Step 2: Fold the circle in half three times to get 8 equal-sized parts.

Step 3: Cut the eight equal-sized parts of the circle on the fold lines.

Step 4: Arrange the pieces as shown.



Step 5: Which quadrilateral does the new figure resemble?

Step 6: Fill in the blanks.

a. The height of the figure is equal to the _____ of the circle.

b. The base of the figure is equal to half the _____ of the circle.

Step 7: How can the area of this figure be found? Create a formula to find the area of the circle.

Step 8: Use the formula to find the area of one coaster described at the beginning of this lesson.

Step 2: For a more accurate parallelogram, have students fold their circle one more time for a total of 16 sections.

Step 4: Make sure students are alternating the curved and pointy parts as they arrange their pieces.

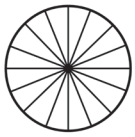
Step 5: parallelogram.

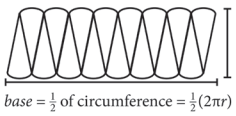
Step 6: Give students time to come up with the answers on their own. **a)** radius **b)** circumference

Step 7: Students may need assistance getting to the final area formula. The steps are written out on page 57, but a teacher-guided explanation may be beneficial. Find the product of the radius and the circumference. Area $= \frac{1}{2}C \cdot r = \frac{1}{2}(2r\pi) \cdot r = r\pi \cdot r = \pi r^2$

Step 8: The area of one coaster is approximately 78.5 cm^2.

The area formula of a circle can be determined by arranging pieces of a circle to form a shape similar to a parallelogram. The height of the parallelogram is the radius of the circle. The length of the base is half the circumference of the circle.





The area of a parallelogram is found by multiplying base times height. When the circle is arranged to form a parallelogram, the area of the circle can be determined by multiplying the base times the height.

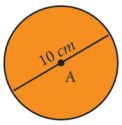
$$Area = base \cdot height$$
$$Area = \tfrac{1}{2}C \cdot r$$
$$Area = \tfrac{1}{2}(2\pi r) \cdot r$$
$$Area = \pi r \cdot r$$
$$Area = \pi r^2$$

AREA OF A CIRCLE

$A = \pi r^2$

The area of a circle is the product of π and the square of the radius.

EXAMPLE 1 **Find the area of the circle. Use 3.14 for π.**



SOLUTION

Write the area formula.	$A = \pi r^2$
Find the length of the radius.	$r = 10 \div 2 = 5$
Substitute the known values.	$A \approx (3.14)(5)^2$
Square 5.	$A \approx (3.14)(25)$
Multiply.	$A \approx 78.5$

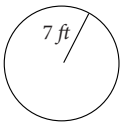
The diameter is given. However, the radius is needed. Divide the diameter by 2 to get the length of the radius.

The area of ⊙A is approximately 78.5 square centimeters.

The circle in **Example 1** is the same size as Maria's and Greg's coasters from the beginning of the Lesson. Each coaster will be approximately 78.5 square centimeters.

Extra Example 1

Find the area of the circle. Use 3.14 for π.



$A \approx 153.86\ ft^2$

Mathematical Practices – A Closer Look

SMP1 The **Explore!** "Examine Circle Areas" is challenging. **Steps 5–8** will make some students frustrated. Encourage students to work together with a partner or group to make sense of the figure in **Step 5** as it relates to parallelograms and parts of circles.

SMP2 In **Exercises 4–9** and **13–14**, students will have to translate the information in a word problem to a figure and an equation in order to answer the question posed. Use **Example 2** to model this process for students.

SMP3 In the **Explore!** students will use their knowledge of parallelogram area and circumference to develop a formula for the area of a circle. Have students explain their reasoning to one another and verify the work of their peers.

SMP5 When using an approximation of π an "approximately equal to" sign (≈) should be used. Label answers with correct units.

Extra Example 2

Peggy knitted a circular blanket. The diameter of the blanket is 3 feet. What is the area of the blanket? Use 3.14 for π. ≈ 7.065 ft^2

EXAMPLE 2

Jaira is having pizza for her birthday party. She can make two small pizzas, each with a radius of 6 inches, or one large pizza with a radius of 12 inches. Which option will give her more square inches of pizza?

SOLUTION

	Two small pizzas	One large pizza
Write the circle area formula.	$A = \pi r^2$	$A = \pi r^2$
Substitute the known values.	$A \approx (3.14)(6)^2$	$A \approx (3.14)(12)^2$
Square the radius.	$A \approx (3.14)(36)$	$A \approx (3.14)(144)$
Multiply.	$A \approx 113.04$	$A \approx 452.16$
Multiply the area of a small pizza by 2 for two small pizzas.	$A \approx 113.04(2) \approx 226.08$	

The large pizza will give Jaira twice as much pizza as two small pizzas.

Up to this point you have used a common estimate for pi, 3.14, for calculations. There will be situations when exact answers are needed for problems involving circles. For example, a manufacturer might program a machine to create a circular tarp with a radius of 3 feet. The exact area of one tarp needs to be calculated in order to find the amount of material used for one tarp. Estimating means the answer is not exact.

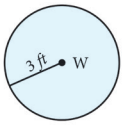
Exact answers are written using the π symbol. A common estimate of pi should not be substituted for pi when an exact answer is required.

Extra Example 3

Ricardo needs to find the exact area of a circular computer chip. The radius of the chip is 7 *mm*. What is the exact area of the chip? $49\pi\ mm^2$

EXAMPLE 3

Find the exact area of ⊙W.



SOLUTION

Write the circle area formula.	$A = \pi r^2$
Substitute known values.	$A = \pi(3)^2$
Square 3.	$A = \pi(9)$
Rewrite with the number before the symbol.	$A = 9\pi$

The exact area of ⊙W is 9π square feet.

Teaching Tips

Some students may multiply by 2 instead of squaring a number. Keep a watchful eye out for this. Multiplying by 2 will find the circumference!

It may be helpful to show the steps needed for simplifying area. For example, $3^2(3.14) = 3 \cdot 3 \cdot (3.14) = 9 \cdot (3.14) = 28.26$.

Explain that circumference is the length of rope needed to go around the edge of a circle, while area is the number of square units that fit inside of the circle.

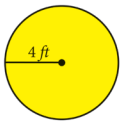
Exact answers can often be found without a calculator and will be given most often in future math classes. However, students may have a difficult time with the concept of leaving a symbol in a solution. Reinforce the fact that 3.14 is an approximation for π. The only exact form of π is the symbol. More approximations of π will be discussed in **Lesson 2.6**.

Remind students to read directions very carefully to find if an exact answer is needed or an estimation.

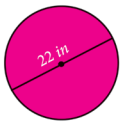
EXERCISES

Find the area of each circle. Use 3.14 for π.

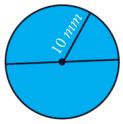
1.



2.



3.



4. Mark set up a sprinkler to water the backyard. The sprinkler waters in a circular motion. The radius of the area covered by the sprinkler is 12 feet. Find the approximate square footage of the watered area.

Theo's Work

$A = \pi r^2$
$A \approx (3.14)(16)^2$
$A \approx (3.14)(32)$
$A \approx 100.48$
$100.48\ in^2$

5. An extra large pizza has a radius of 16 inches. Theo miscalculated the area of the pizza in his work at left. Identify his error and determine the actual area of the pizza.

6. A circular tablecloth has a diameter of 14 feet. What is the area of the tablecloth?

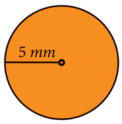
7. A lighthouse beam reaches 21 miles in all directions. How many square miles does the light cover? Round the answer to the nearest square mile.

8. The diameter of a classroom clock face is 13 inches. Find the area of the clock face.

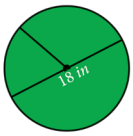
9. Lacey plans to put another window in her bedroom. She needs to decide whether to put in two small windows, each with a 1 foot radius, or one large window with a 2 foot radius. She wants as much sunlight as possible in her room. Which option should she use? Show all work necessary to justify your answer.

Calculate the exact area of each circle.

10.



11.



12. A circle with a diameter of 14.4 feet.

13. A milling machine is programmed to cut a hole with a radius of 8 millimeters. The computer program requires the programmer to enter the area of the hole. Find the exact area of the hole.

14. A circle has a radius of 1 inch. Find the exact area of the circle.

15. One circle has a diameter of 5 inches. Another circle has a radius of 2.5 inches. How do the areas of the circles compare? Use words and/or numbers to show how you determined your answer.

16. Sandra found the area of a circle to be exactly 16π square centimeters. If a circle has a larger area than Sandra's circle, what could the length of the radius be? Explain how you know your answer is correct.

Focused Assignment

Exercises 1–7, 13–27 odd

Exercise Answers

1. ≈ 50.24 ft^2

2. ≈ 379.94 in^2

3. ≈ 314 mm^2

4. ≈ 452.16 ft^2

5. Theo multiplied 16 by 2 instead of squaring 16. The area of the pizza is about 803.84 in^2.

6. ≈ 153.86 ft^2

7. ≈ 1,385 square miles

8. ≈ 132.665 in^2

9. The large window with a radius of 2 feet. See student work (e.g., the two small windows have an area of about 6.28 ft^2 and the one large window has an area of about 12.56 ft^2).

10. $25\pi\ mm^2$

11. $81\pi\ in^2$

12. $51.84\pi\ ft^2$

13. $64\pi\ mm^2$

14. $\pi\ in^2$ or $1\pi\ in^2$

15. The circles have equal areas because they both have a radius of 2.5 inches; see student work.

16. Answers may vary. The radius must be greater than 4 *cm*. See student work.

17. a. ≈ 18 inches
b. ≈ 254.34 square inches

18. 9π square feet; See student work.

19. ≈ 4.91 m^2; See student work.

20. 8 *in*

21. a. 8π cm^2
b. 4π cm^2
c. 2π cm^2
d. $\frac{16}{3}\pi$ cm^2 or $5\frac{1}{3}\pi$ cm^2

22. H

23. two of the following: $\overline{HC}$, $\overline{HI}$ or $\overline{HP}$

24. one of the following: ∠IHC, ∠IHP or ∠CHP

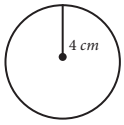
25. $\overline{ZU}$

26. $\overline{CP}$

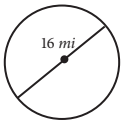
27. ⊙H

Exit Problems

1) Use 3.14 to find the area of each circle.

a) 

≈ 50.24 cm^2

b) 

≈ 200.96 mi^2

2) What is the exact area of a circle with a radius of 5 inches? 25π in^2

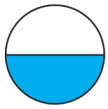
17. A dart board has a circumference of 56.52 inches.
a. Find the diameter of the dartboard. Use 3.14 for π.
b. Find the approximate area of the dart board.

18. The exact circumference of a circle is 6π feet. Find the exact area of the circle. Show all work necessary to justify your answer.

19. The circumference of a circle is about 7.85 meters using 3.14 for π. Find the area of the circle. Round the answer to the nearest hundredth and use mathematics to justify your answer.

20. The area of a circle is 16π in^2. What is the diameter?

21. Each circle has a radius of 4 *cm*. Find the exact area of each shaded region, given that the regions in each circle are equal in size.

a. 
b. 
c. 
d. 

REVIEW

Use the diagram at right.

22. Name the center.

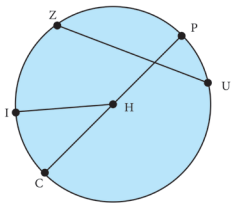
23. Identify two radii.

24. Identify a central angle.

25. Name the shortest chord.

26. Name the longest chord.

27. What is the name of the circle?



Tic-Tac-Toe ~ Ferris Wheels

Write a research paper about the Ferris wheel. It must be at least one page in length. Include the following:
- facts about the first Ferris wheel;
- information about 2 additional Ferris wheels;
- measurements for diameter, radius and circumference;
- a list of resources used to find the information;
- a diagram of one Ferris wheel you researched labeled with the measurements.

Communication Prompts

1. What is the same about the formulas for area and circumference of a circle? What is different about the formulas for area and circumference of a circle?

2. How will you remember that $2\pi r$ is the formula for circumference and πr^2 is the formula for area?