# QUADRATIC FUNCTIONS IN FACTORED FORM

## LESSON 3.3

Use factored form to find the zeros of a quadratic function and graph the function.

There are three common forms used to write quadratic functions. So far, you have learned two of them:

- Vertex Form: $f(x) = a \cdot (x - h)^2 + k$
- General Form: $f(x) = ax^2 + bx + c$

The third form used to write quadratic functions is called factored form.

**Factored Form of a Quadratic Function**

A quadratic function is in factored form when written

$$f(x) = a(x - r_1)(x - r_2)$$

where $a \neq 0$.

**EXPLORE!** **FINDING ZEROS**

A graphing calculator can help you explore properties of different types of functions. In this **Explore!**, you will use a graphing calculator to determine how to find key information about a quadratic function in factored form.

**Step 1:** Set your viewing window to $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$. Graph the function $y = (x - 2)(x + 4)$. Sketch what you see on your calculator screen. Where does the graph cross the $x$-axis?

**Step 2:** For each function, sketch a simple graph of the function by graphing the functions on your calculator. List the two values for each function which describe where the graph crosses the $x$-axis. These are called the $x$-intercepts.

**a.** $y = (x + 6)(x + 1)$ **b.** $y = -(x - 1)(x - 5)$ **c.** $y = x(x + 4)$

**d.** $y = 2(x - 3)(x - 7)$ **e.** $y = 4(x + 1)(x - 1)$ **f.** $y = -2(x + 8)(x + 4)$

**Step 3:** How do the $x$-intercepts relate to the equation in this form?

**Step 4:** The $x$-intercepts of quadratic functions are often called "zeros". Explain why you think they are called the "zeros" of the equation.

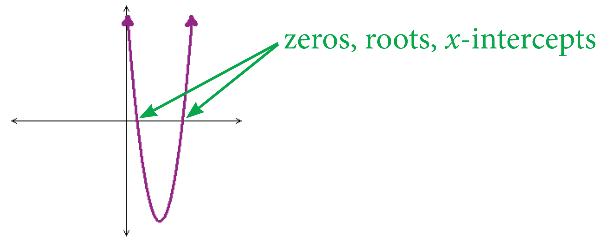
**Step 5:** How does the value and the sign of the coefficients (the number in front of the parentheses) affect the graphs?

## EXPLORE! (CONTINUED)

**Step 6:** Use what you learned in **Steps 1-5** to PREDICT what the following graphs will look like. Use your calculator to check your answers.

**a.** $y = (x + 9)(x + 2)$ **b.** $y = 2(x + 3)(x - 1)$ **c.** $y = -x(x - 6)$

The $x$-intercepts of a quadratic function are also called the **zeros** or **roots** of the quadratic function. All of these terms are used to tell where a graph crosses the $x$-axis.




The $x$-intercepts can always be found by setting the output value of a function to 0. When a quadratic function is written in factored form, you can quickly determine the zeros using the **Zero Product Property**. The Zero Product Property states that if a product of two factors is equal to zero, then one or both of the factors must be zero.

## EXAMPLE 1

**Find the zeros of each quadratic function.**

**a.** $f(x) = (x + 7)(x - 3)$ **b.** $y = -2(x + 1)(x + 10)$

### SOLUTIONS

**a.** Set the equation equal to zero. $0 = (x + 7)(x - 3)$

The Zero Product Property says at least one of the factors must equal 0, so set each factor equal to zero. $0 = x + 7$ $0 = x - 3$

Solve each equation for $x$.

$$\begin{array}{rr} 0 = x + 7 & 0 = x - 3 \\ \underline{-7 \quad -7} & \underline{+3 \quad +3} \\ -7 = x & 3 = x \end{array}$$

The zeros of the function are (−7, 0) and (3, 0).

**b.** Set the equation equal to zero. $0 = -2(x + 1)(x + 10)$

Set each factor equal to zero. $0 = x + 1$ $0 = x + 10$

−2 ≠ 0 so it is not listed.

Solve each equation for $x$.

$$\begin{array}{rr} 0 = x + 1 & 0 = x + 10 \\ \underline{-1 \quad -1} & \underline{-10 \quad -10} \\ -1 = x & -10 = x \end{array}$$

The zeros of the function are (−1, 0) and (−10, 0).

Often there is a front coefficient, $a$, on quadratic equations in factored form. Just like with other forms, when $a$ is positive, the parabola opens upward. When $a$ is negative, the parabola opens downward.

The value of $a$ also tells you whether the graph has been stretched or shrunk compared to the parent function.

$$f(x) = a(x - r_1)(x - r_2)$$

## Graphing a Quadratic Function in Factored Form

A quadratic function can be graphed from factored form by finding three key points on the graph.

1. Find the $x$-intercepts using the Zero Product Property.
2. The axis of symmetry occurs exactly halfway between the two $x$-intercepts. Average the two $x$-intercepts to find the equation for the axis of symmetry.
3. Substitute the $x$-value of the vertex (from the axis of symmetry) into the original function to find the $y$-value of the vertex.
4. Graph the three points (two $x$-intercepts and vertex). Connect the points with a smooth curve. You may also create a table using points on either side of the vertex.

**EXAMPLE 2** **Graph the quadratic function $y = (x - 2)(x - 6)$.**

**Solution**

To find the $x$-intercepts, set the equation equal to zero. $0 = (x - 2)(x - 6)$

Using the Zero Product Property, set each factor equal to zero. $0 = x - 2 \qquad 0 = x - 6$

Solve each equation for $x$.

$$\begin{array}{rr} 0 = x - 2 & 0 = x - 6 \\ \underline{+2 \quad\quad +2} & \underline{+6 \quad\quad +6} \\ 2 = x & 6 = x \end{array}$$

The $x$-intercepts of the function are (2, 0) and (6, 0).

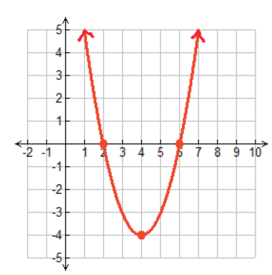
Find the axis of symmetry by averaging the two $x$-intercepts. Average by adding and dividing by two. $\frac{2 + 6}{2} = 4$

The equation for the axis of symmetry is $x = 4$. Substitute $x = 4$ into the original function. $y = (4 - 2)(4 - 6)$

Evaluate to find the $y$-coordinate. $y = 2(-2) = -4$

The vertex is at (4, −4).

Graph the three points ($x$-intercepts and vertex) and connect it with a smooth curve.




Unlike vertex and general form, you do not need to create an input-output table to find additional points on the curve to graph the parabola. If you want more points to make your graph more accurate, you can substitute additional input values into the function.

## EXERCISES

**Find the zeros of each quadratic function.**

**1.** $f(x) = (x - 5)(x - 9)$

**2.** $m(x) = (x - 1)(x + 8)$

**3.** $g(x) = (x + 1)(x + 4)$

**4.** $y = (x - 5)(x + 2)$

**5.** $p(x) = -x(x + 6)$

**6.** $y = -3(x - 4)(x - 5.5)$

**7.** How do the zeros of a quadratic function help you graph the function?

**Determine the coordinates of the vertex for each quadratic function and whether the parabola has a maximum or a minimum.**

**8.** $f(x) = (x - 3)(x - 9)$

**9.** $g(x) = 2(x + 4)(x - 2)$

**10.** $y = -x(x + 5)$

**11.** A bridge follows the path described by the function $h(x) = -0.25(x - 48)(x - 184)$ where $h(x)$ describes the height of the bridge and $x$ is the distance from the nearest building (both in meters).

**a.** How far from the building does the bridge touch the ground? Hint: there will be two answers.

**b.** How high is the bridge at its tallest point?

**12.** A parabola has an axis of symmetry at $x = 6$ and one of the $x$-intercepts is at (9, 0). Where is the other $x$-intercept? Explain how you know your answer is correct.

**13.** A parabola has an axis of symmetry at $x = -1$ and one of the $x$-intercepts is at $x = 4$. Where is the other $x$-intercept? Explain how you know your answer is correct.

**Graph each quadratic function. Clearly mark the *x*-intercepts and the vertex on the graph.**

**14.** $f(x) = (x - 3)(x - 9)$

**15.** $y = (x + 2)(x - 2)$

**16.** $g(x) = 4(x + 7)(x + 9)$

**17.** $w(x) = 3(x + 1)(x - 1)$

**18.** $y = -(x - 6)(x + 2)$

**19.** $h(x) = -2x(x - 5)$

**20.** Write a quadratic function for a parabola that has $x$-intercepts of 4 and −2.

**21.** Write a quadratic function for a parabola that has $x$-intercepts of 3 and 7 with a vertex at (5, 4).

**22.** Write a quadratic function for a parabola that has $x$-intercepts of −1 and 3 with a vertex at (1, −12). Use words and/or numbers to show how you determined your answer.

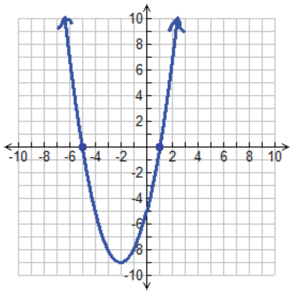
**23.** Write three different quadratic functions in factored form that have an axis of symmetry of $x = 2$ but have different $x$-intercepts. Use words and/or numbers to show how you determined your answer.

**24.** Does the $a$ value in a factored form equation affect the value of the $x$-intercepts? Explain your reasoning.

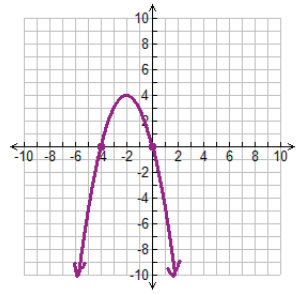
## REVIEW

**Write the equation of each graph in factored form.**

**25.**




**26.**




**Describe how each new function is related to the parent function $f(x) = x^2$.**

**27.** $m(x) = (x + 1)^2$

**28.** $w(x) = -x^2$

**29.** $f(x) = (x - 2)^2 - 6$

**30.** $n(x) = \frac{1}{3}\left(x + \frac{5}{3}\right)^2$

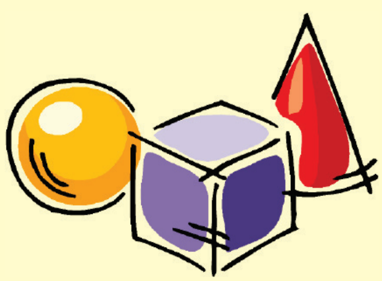
**31.** $g(x) = -6x^2$

**32.** $p(x) = 0.8(x - 1)^2 + 5$

**33.** Fill in numerical values for $a$, $h$ and $k$ in the equation $y = a(x - h)^2 + k$ to make each statement true.

- **a.** A parabola that has a vertex at $(-1, -5)$ and an $a$-value of 3.
- **b.** A parabola that has a vertex at $(6, 0)$ and opens downward.
- **c.** A parabola that has a minimum of $(-4, -10)$ and is shrunk by a factor of 0.2.
- **d.** A parabola that has an axis of symmetry at $x = -3$ and opens upward.

## Tic-Tac-Toe ~ Polynomial Prefixes



| $x^2$ | $4x^2 - 3x$ | $x^3 + 5x - 2$ |
|---|---|---|
| MONOMIAL | BINOMIAL | TRINOMIAL |

The word "**polynomial**" comes from poly- (meaning "many") and -nomial (in this case meaning "term") so it says "many terms". Look at the vocabulary in the boxes above. These are special names for polynomials with 1, 2 or 3 terms. The prefixes (like "poly" in polynomial) give details about the number of terms in each expression.

Create a poster or other type of display that would help other students in your class remember the names of the different polynomials based on their prefixes. Divide the display into four regions. Put one of these words at the top of each region:

Monomial
Binomial
Trinomial
Polynomial

Connect these prefixes to other words you know that use a similar prefix. Include drawings and/or photographs of objects that share a common prefix.