TRANSFORMATIONS AND CONGRUENCE

LESSON 3.5

Understand the relationship between the pre-image and the image of a transformation.

Two figures are congruent if they are the exact same shape and the exact same size. Figures are similar if they are the same shape but different sizes.

In the **Explore!** you will draw the image of a triangle given a transformation and then determine if the image is congruent or similar to the original (or pre-image) triangle.

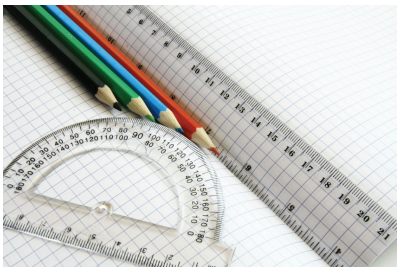
EXPLORE! CONGRUENT OR SIMILAR?

Step 1: List four types of transformations on four separate lines of your paper.

Step 2: Each type of transformation forms an image that is either similar or congruent to the original figure. Make a conjecture (an educated guess) for each transformation you listed in **Step 1** about whether the image will be congruent or similar to the original figure.

Step 3: Use ΔABC as the pre-image for **Steps 4–7**: A(2, 5), B(4, 3) and C(3, 0). Use graph paper to make five coordinate planes (each from −10 to 10 on both the *x*- and *y*-axes). Graph ΔABC on the first coordinate plane.

Step 4: One type of transformation is a translation. On the second coordinate plane, graph ΔA'B'C' by shifting ΔABC 3 units left and 2 units up. What do you notice about the image compared to the pre-image? Use the word "similar" or "congruent" in your observation.



Step 5: Another type of transformation is a reflection. On the third coordinate plane, graph ΔA'B'C' by reflecting ΔABC over the *x*-axis. What do you notice about the image compared to the pre-image? Use the word "similar" or "congruent" in your observation.

Step 6: A dilation is another type of transformation. On the fourth coordinate plane, graph ΔA'B'C' using a dilation with a scale factor of 2. What do you notice about the image compared to the pre-image? Use the word "similar" or "congruent" in your observation.

Step 7: The last type of transformation is a rotation. On the last coordinate plane, graph ΔA'B'C' by rotating ΔABC 180° clockwise about the origin. What do you notice about the image compared to the pre-image? Use the word "similar" or "congruent" in your observation.

Step 8: Compare your observations with your original conjectures. Did your work in **Steps 4–7** support your conjectures? If not, rewrite your conjecture(s) to match your work.

Congruency and Similarity with Transformations

Under a translation, reflection or rotation, the original figure and its image are congruent.

Under a dilation, the original figure and its image are similar.

EXAMPLE 1

Name the type of transformation given by each rule. Then state if the image will be congruent or similar to its pre-image.

a. $(x, y) \rightarrow (x - 3, y + 4)$ **b.** $(x, y) \rightarrow (4x, 4y)$

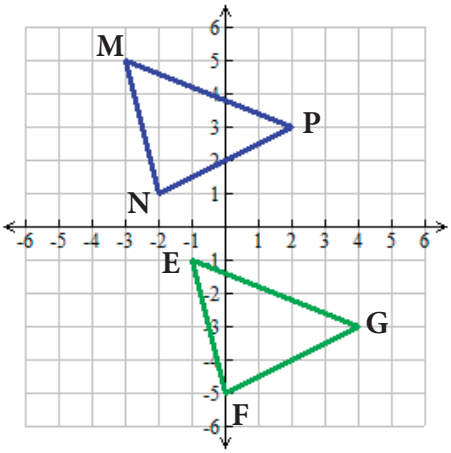
Solutions

a. $(x - 3)$ means translate 3 units left and $(y + 4)$ means translate 4 units up. This is a translation. Translations form congruent figures.

b. A transformation that multiplies the coordinates of a figure by a scale factor (in this case, 4), is a dilation. Dilations form similar figures.

EXAMPLE 2

ΔEFG was formed by a single transformation of ΔMNP.



a. Are ΔEFG and ΔMNP congruent or similar?
b. Write a transformation rule that maps ΔMNP onto ΔEFG.

Solutions

a. ΔEFG and ΔMNP are congruent because they are the same shape and same size.
b. ΔMNP is moved 2 units to the right and 6 units down to make ΔEFG. This transformation rule can be written $(x, y) \rightarrow (x + 2, y - 6)$.

EXERCISES

Write the transformation rule for each transformation. State whether the transformation creates similar or congruent figures.

1. A reflection over the y-axis

2. A translation up 4 units

3. A dilation with a scale factor of 3

4. A reflection over the x-axis

5. A translation 2 units left and 8 units down

6. A dilation with a scale factor of 0.25

Name the type of transformation given by each rule. State whether the transformation creates similar or congruent figures.

7. $(x, y) \rightarrow (7x, 7y)$

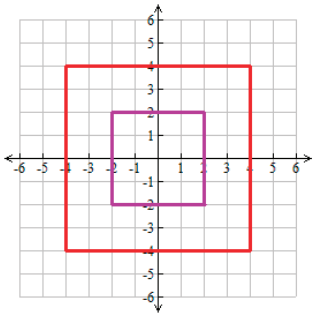
8. $(x, y) \rightarrow (-x, -y)$

9. $(x, y) \rightarrow (-x, y)$

10. $(x, y) \rightarrow (x + 5, y - 4)$

11. Which type of transformation does not always create a congruent figure?

12. Jacob shifted a figure five units down and two units left. He predicted that the resulting figure would be similar, but not congruent, to his original shape. Do you agree or disagree? Explain your reasoning.



13. The red square at the left was graphed first. A transformation was performed on the red square to form the purple square.

- **a.** Write a transformation rule that maps the red square onto the purple square.
- **b.** What type of transformation is this?
- **c.** Are the figures similar or congruent?

14. Write a transformation rule that creates an image congruent to its pre-image.

15. Write a transformation rule that creates a figure similar to its original.

16. Use the triangle formed by the vertices P(−3, 4), A(0, 5) and N(2, 1).

- **a.** ΔP'A'N' is a triangle formed by using the transformation rule $(x, y) \rightarrow (x, -y)$. Graph ΔP'A'N'.
- **b.** What type of transformation is ΔPAN → ΔP'A'N'?
- **c.** Are ΔPAN and ΔP'A'N' congruent or similar?

17. Nachelle wrote the transformation rule $(x, y) \rightarrow (2x, 3y)$. She says it will produce figures that are similar to one another but not congruent. Do you agree or disagree with Nachelle's statement? Support your answer by graphing an original figure and its image using the transformation rule.

18. Jina stated that there is one dilation that creates congruent figures. Is she correct? If so, what scale factor creates congruent images?

19. Rectangle ABCD is transformed to form rectangle EFGH. The coordinates for ABCD are A(2, 3), B(2, 5), C(5, 5) and D(5, 3). The coordinates for EFGH are E(−2, 3), F(−2, 5), G(−5, 5) and H(−5, 3). Describe how the original figure and the new figure are related (transformation type and congruence/similarity).

20. Jeff said he could reflect a figure and then translate the reflected figure. He said the final image would be similar, but not congruent, to the pre-image. Is he correct? Show all work necessary to justify your answer.

REVIEW

Find the missing side length in each right triangle. If necessary, round to the nearest tenth.

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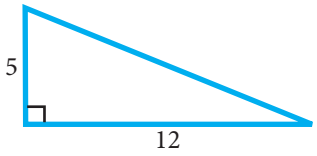

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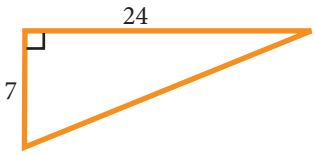

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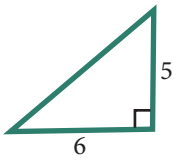

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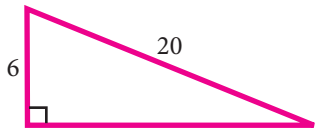

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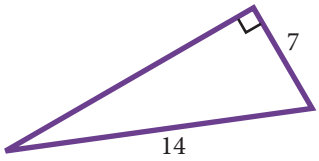

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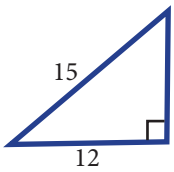

Tic-Tac-Toe ~ Kaleidoscope

The word "kaleidoscope" means to examine beautiful forms. Sir David Brewster invented the kaleidoscope in 1815. When it was manufactured, 200,000 kaleidsoscopes were sold in London and Paris. Research how a kaleidoscope is made and works. Write a booklet explaining how to make a kaleidoscope and explain how it works.

Tic-Tac-Toe ~ Transformation Recap

Create a booklet which gives the highlights of each of the four types of transformations. Include definitions, examples, transformation rules and information about whether the image would be congruent or similar to the pre-image for each type of transformation.

In the last page of your booklet, write a five-to ten-problem "Quick Check" that others could use to test their knowledge of the different transformations. Include the answers on the back cover of your booklet.