

USING PROBABILITY TO PREDICT

LESSON 4.2



Predict an outcome using experimental and theoretical probability.

John predicted who was going to win the class election for president. He used a poll. He asked 30 seventh graders who they were going to vote for. Twenty of those asked said they were going to vote for Mike. This meant $\frac{2}{3}$ of those asked were going to vote for Mike. Based on this experimental probability, John predicted Mike would win.



Cheryl wanted to know how many salmon make it through the dams on the Columbia River. She tagged a few salmon down river. She kept track of the tagged salmon that made it through the dams. She used her information to predict the overall number of salmon that made it through the dams and up the river.

Experimental probability is used to make predictions. John used it to predict who would win a class election. Cheryl used experimental probability to predict how many salmon made it up the river. You will use capturing and recapturing beans to make a prediction in the **Explore!**

EXPLORE!

The white beans in this experiment represent salmon in the Columbia River.

Step 1: Take out a handful of white beans and replace them with black beans. The black beans represent tagged salmon. Record how many black beans are put into the bag with the white beans.

Number of tagged (black) beans: _____

Step 2: Mix the white and black beans together. Take out a small handful of beans. Record the total number of beans in the sample and how many are tagged.

Number of tagged (black) beans in handful: _____

Total number of beans in handful: _____

Step 3: Use the proportion below to estimate the total number of beans (x) that are actually in the bag.

$$\frac{\text{tagged beans in handful}}{\text{total beans in handful}} = \frac{\text{tagged beans in bag (number from Step 1)}}{\text{total beans in bag (} x \text{)}}$$

Use your information from **Steps 1 and 2** to fill in the proportion and solve for x .

$$\frac{\quad}{\quad} = \frac{\quad}{x}$$

Round the answer to the nearest whole number. Record it in the chart in **Step 4**.

CAPTURE / RECAPTURE



Step 4: Repeat **Steps 2 and 3** four more times to get five estimates for the number of beans originally in the bag. Copy and complete the chart to record your information.

Handful	Number of tagged beans in handful	Total beans in handful	Estimate of beans actually in the bag: $\frac{\text{tagged beans in handful}}{\text{total beans in handful}} = \frac{\text{tagged beans in bag (Step 1)}}{\text{total beans in bag } (x)}$
Sample 1			$\frac{\quad}{\quad} = \frac{\quad}{x} \quad x = \quad$
Sample 2			$\frac{\quad}{\quad} = \frac{\quad}{x} \quad x = \quad$
Sample 3			$\frac{\quad}{\quad} = \frac{\quad}{x} \quad x = \quad$
Sample 4			$\frac{\quad}{\quad} = \frac{\quad}{x} \quad x = \quad$
Sample 5			$\frac{\quad}{\quad} = \frac{\quad}{x} \quad x = \quad$

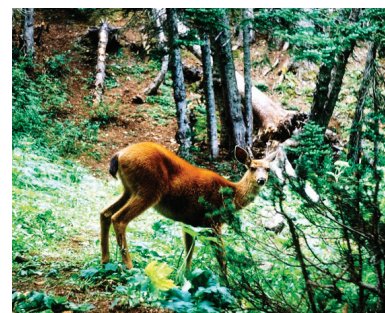
Step 5: Find the average of your five estimates in the chart above. Do this by finding the sum of all five estimates. Divide the sum by five (the number of estimates).

Step 6: Count and record all of the beans in the bag (white and black beans). Compare your count with your estimates and average.

Step 7: What do you think would happen to the average if you used 100 samples (handfuls) to estimate the actual number of beans in the bag?

EXAMPLE 1

Park Rangers estimated the number of deer in a section of Yellowstone National Park. The rangers tagged 20 deer in campgrounds and near hiking trails. The following week they recaptured 300 deer, 15 of which were tagged. Estimate the total number of deer in this section of Yellowstone National Park.



SOLUTION

Let x = total number of deer. Write a proportion using the number of tagged deer recaptured to the total number of deer recaptured as one ratio.

$$\frac{\text{number of tagged deer recaptured}}{\text{total number of deer recaptured}} = \frac{\text{number of tagged deer}}{\text{total number of deer}} = \frac{15}{300} = \frac{20}{x}$$

Use cross products to solve.

$$\begin{aligned} 15x &= 6000 \\ x &= 400 \end{aligned}$$

There are about 400 deer in this section of Yellowstone National Park.

EXAMPLE 2

Today at batting practice, Lou hit 12 home runs out of the 40 pitches thrown to him. Use experimental probability to predict how many home runs he will hit tomorrow if he gets 30 pitches.

SOLUTION

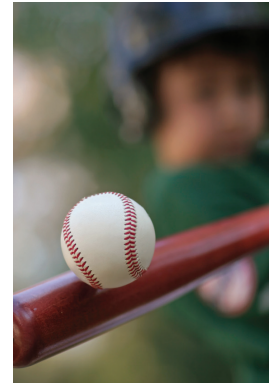
Let x = number of home runs hit tomorrow. Write a proportion using Lou's experimental probability as one ratio.

$$\frac{\text{number of home runs Lou hit}}{\text{number of pitches thrown}} = \frac{\text{number of home runs hit tomorrow}}{\text{number of pitches thrown tomorrow}}$$

Write a proportion.
$$\frac{12}{40} = \frac{x}{30}$$

Use cross products to solve.
$$40x = 360$$
$$x = 9$$

He should hit about 9 home runs tomorrow.



EXERCISES

Show all work necessary to justify your answer for each Exercise.



1. Sarah surfs with her friends at Virginia Beach. Her friend, Mark, caught 8 waves out of 20 possible waves to ride in to shore. They stay out in the water for 50 more waves. Predict how many of the 50 waves Mark will catch.

2. A professional basketball player practiced shooting free throws before a game. He made 12 of 15 shots. Predict how many free throws he will make in the game if he has 5 opportunities.

3. Aiden flipped a coin 20 times. It landed heads 12 times.
- Use experimental probability to find $P(\text{heads})$.
 - Use Aiden's experimental probability to predict how many heads he will get if he flips the coin 30 times.
 - Find $P(\text{heads})$ when flipping a coin once using theoretical probability.
 - Use theoretical probability to predict how many heads Aiden will get if he flips the coin 20 times.
 - Use theoretical probability to predict how many heads Aiden will get if he flips the coin 30 times.
4. Casey polled 50 seventh grade students. He asked who they planned to vote for as class president. Thirty students said they would vote for Hillary. Twenty said they would vote for Marcus.
- Find $P(\text{a student votes for Hillary})$ using experimental probability.
 - Use this probability to predict how many seventh graders in their class of 450 students will vote for Hillary.
 - Would you predict a victory for Hillary? Explain your reasoning.



5. Park Rangers wanted to estimate the total number of wild mustangs in the park. They tagged 30 wild mustangs and sent them back to the herd. They recorded the mustang population once a month for the next three months as shown below.

	Month 1	Month 2	Month 3
Tagged mustangs in sample	8	10	15
Total mustangs in sample	100	150	200

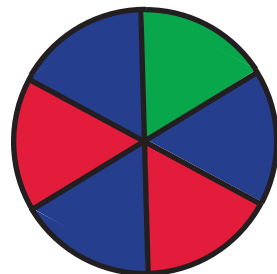


- Estimate the size of the wild mustang population for each month.
 - Find the average of the three estimates in **part a**.
 - The rangers combined the three months into one large sample so they could consider a sample of $100 + 150 + 200 = 450$ mustangs. Estimate the size of the wild mustang population using the combined sample.
6. One hundred twenty-five townspeople were interviewed at random. Forty of the interviewed townspeople were from the middle school. The middle school has 800 students. Estimate the size of the town.
7. For the the 2012 election, 39% of a state's voters were registered as Democrats and 36% were registered as Republicans.
- Find $P(\text{a registered voter was a Democrat})$.
 - Find $P(\text{a registered voter was a Republican})$.
 - If 2,500,000 people voted during the election in that state, how many would you predict voted for Barack Obama (Democrat)?
 - If 2,500,000 people voted during the election in that state, how many would you predict voted for Mitt Romney (Republican)?
 - What percentage of people in that state did not declare affiliation with the Democrats or Republicans in 2012?



8. The Coast Guard was concerned about the growing number of harbor seals in La Jolla. They tagged 15 seals and sent them back into the bay. After two months they checked a sample of 40 seals. Three of the seals in the sample were tagged. Estimate how many seals are in La Jolla.

9. Use the spinner to the right.
- Ross spun the spinner 12 times. Estimate the number of times he spun green.
 - Ross spun the spinner 18 times. Estimate the number of times he spun red.
 - Ross spun the spinner several times and spun blue 15 times. Estimate how many times he spun the spinner.



10. a. Rob plans to roll a number cube 24 times. He predicts he will roll a 5 six times because there is a $\frac{1}{6}$ chance he will roll a 5. Is he correct? Use mathematics to justify your answer.
- b. Rob rolls the number cube 24 times. It lands on 5 six times. Does this seem reasonable? Explain your reasoning.

REVIEW

Write each percent as a fraction.

11. 25%

12. 60%

13. 100%

14. Sixty out of 800 people interviewed prefer romantic comedy movies. Find the probability that a person chosen at random prefers romantic comedies.

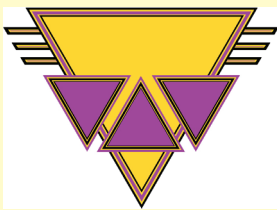
15. After flipping a coin 60 times, Mary's coin landed heads 5 times. Is it a fair coin? Explain your reasoning.

16. Alvin took a group hiking on a trail in the Black Hills. The group hiked 3.2 miles in the first two hours. They hiked for five more hours at the same rate. How far did they hike over the last five hours?

17. Sally drank 60 ounces of water in 8 hours. If she continued drinking water at this rate, how much water would she drink in 15 hours?



Tic-Tac-Toe ~ GEOMETRIC PROBABILITY

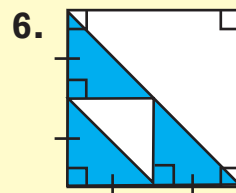
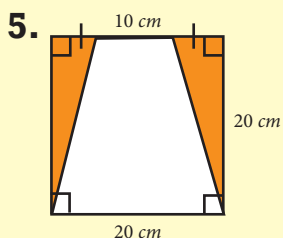
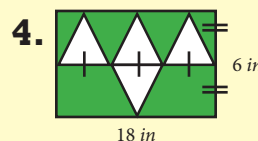
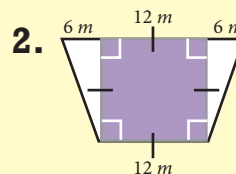
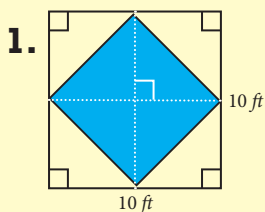


You can find the probability that a dart that lands on a shape will land in a shaded region. Find the area of each shaded region and then find the area of the outside shape which represents the shape of the dart board.

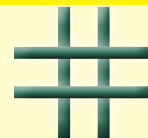


$$P(\text{shaded region}) = \frac{\text{area of shaded region}}{\text{area of outside shape}}$$

Find the geometric probability a dart lands in the shaded region.



Tic-Tac-Toe ~ HITTING PREDICTIONS



Batting averages for baseball players are computed by finding the ratio of the number of hits a batter has to his number of times at bat.

To find Hank Aaron's batting average, find the ratio of his hits to his at bats.

$$\text{Batting Average} = \frac{\text{number of hits}}{\text{number of at bats}} = \frac{3771}{12364}$$

Although $\frac{3771}{12364}$ is the ratio that describes Aaron's batting average, batting averages are always given as decimals rounded to the nearest thousandth.

$$\text{Aaron's batting average} = \frac{3771}{12364} = 0.3049983... \approx 0.305$$

This is read as "Hank Aaron's batting average is 305".

Pretend Hank Aaron played in one more game and had 4 at bats. How many hits would you predict he would have had in the game?

Aaron's batting average is also experimental probability. You would predict he had a 0.305 chance of getting a hit (30.5% chance). Use the percent equation to find the answer to how many hits you would predict.

$$x = 0.305 \cdot 4$$

$$x = 1.22$$

You would predict 1 hit.

1. Suppose Alex Rodriguez finishes the baseball season with a batting average of 0.333 because he has recorded 180 hits in 540 at bats. Use his season's performance as a predictor of success in the World Series. How many hits would you estimate he would hit in six World Series games that give him four at bats each?
2. Suppose each of the players in the chart below are given the chance to bat in six World Series games and get four at bats in each game. Predict the number of hits each player would have in the games.

	Ty Cobb	Reggie Jackson	Mickey Mantle	Tony Gwynn	Jackie Robinson	Edgar Martinez
Batting Average	0.366	0.262	0.298	0.338	0.311	0.312
Number of Hits Predicted						

Source: www.infoplease.com

3. In 22 seasons, Barry Bonds had a lifetime batting average of 0.298. He played in one World Series where he had 8 hits in 17 at bats. Make at least two observations about his World Series performance. Use data to support your observations.